

4.3 + 4.4 Sections

Quotient Rule

Quotient Rule → Chain Rule

Quotient Rule

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

Example: $\frac{(3x-7)^8}{(5x+2)^{12}} = h(x)$

3 main Rules:

Power = If you have a power
 Quotient = If you have division
 Product = If you have multiplication

Step 1

Using Quotient Rule
 (Find f and g)
 $f = (3x-7)^8$

$$g = (5x+2)^{12}$$

Step 2

Find $f' = 8(3x-7)^7 * 3$
 $= 24(3x-7)^7$

↑
 the inside
 $(3x-7)$
 derivative.

Find $g' = 12(5x+2)^{11} * 5$
 $= 60(5x+2)^{11}$

↑
 The inside
 $(5x+2)$
 derivative

Step 3

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2}$$

so

$$f' = 24(3x-7)^7$$

$$g = (5x+2)^{12}$$

$$g' = 60(5x+2)^{11}$$

$$f = (3x-7)^8$$

$$\text{so } \frac{24(3x-7)^7(5x+2)^{12} - 60(5x+2)^{11}(3x-7)^8}{((5x+2)^{12})^2}$$

Step 4

Simplify denominator, so $((5x+2)^{12})^2 = (5x+2)^{24}$

$$\frac{24(3x-7)^7(5x+2)^{12} - 60(5x+2)^{11}(3x-7)^8}{(5x+2)^{24}}$$

Step 5

Find common factors and pull out

$$\frac{12(3x-7)^7(5x+2)^{11} [2(5x+2) - 5(3x-7)]}{(5x+2)^{24}}$$

} End of Calculus, now Algebra

Step 6

$$\text{Simplify: } \frac{12(3x-7)^7(5x+2)^{11}(-5x+39)}{(5x+2)^{24-13}} = \frac{12(3x-7)^7(-5x+39)}{(5x+2)^{13}}$$

Step 7 Final answer:
$$\frac{12(3x-7)^7(-5x+39)}{(5x+2)^{13}}$$

Using Product Rule:

$$h(x) = \underbrace{(3x-7)^8}_{(f)} \underbrace{(5x+2)^{-12}}_{(g)}$$

Step 1 Choose an (f) and (g)

Step 2 $f = (3x-7)^8$
 Find $f' = 8(3x-7)^7 * 3 = 24(3x-7)^7$

$g = (5x+2)^{-12}$
 $g' = -12(5x+2)^{-13} * 5 = -60(5x+2)^{-13}$

Step 3 Cross multiply

$$(fg)' = f'g + g'f = 24(3x-7)^7(5x+2)^{-12} + (-60(5x+2)^{-13})(3x-7)^8$$

Step 4

Side example

Step 1 $x^{-3} + x^{-4}$ option: $\frac{1}{x^3} + \frac{1}{x^4} = \frac{1}{x^3} \left(1 + \frac{1}{x}\right)$

Step 2

Find LCD: $\left[\frac{1}{x^3} + \frac{1}{x^4} \right] = \frac{x}{x^4} + \frac{1}{x^4}$

or $\frac{1}{x^4} \cdot (x+1)$ Step 3

Side example #2

$x^{-3} + x^{-4}$, so $x^{-4}(\underline{x^{-4}x^1} + \underline{x^{-4}})$
 $= x^{-3}$

$x^{-3} + x^{-4}$
 $x^{-4}x^1 + x^{-4} = x^{-4}(x+1) = \frac{1}{x^4}(x+1)$

Side example #3

$x^{-12} + x^{-13}$ { Take out x^{-13} }

$x^{-13} \cdot x^1 + x^{-13} = x^{-13}(x+1)$

Formula:

$$(fg)' = f'g + g'f$$

Equation:

$$\begin{aligned} (f'g) & 24(3x-7)^7(5x+2)^{-12} \text{ less} \\ (g'f) & 60(5x+2)^{-13}(3x-7)^8 \end{aligned}$$

$$= 24(3x-7)^7(5x+2)^{-12} - 60(5x+2)^{-13}(3x-7)^8$$

Step 4 Take out common factors

$$12(3x-7)^7(5x+2)^{-13} \underbrace{[2(5x+2) - 5(3x-7)]}$$

Step 5

Simplify by distributing

$$\begin{aligned} & 10x+4-15x+35 \\ & = -5x+39 \end{aligned}$$

Step 6 Clean it up!!

$$\frac{12(3x-7)^7(-5x+39)}{(5x+2)^{13}}$$